On the other hand, in the presence of an undistributed solute, Marangoni effects are very possibly the mechanism by which the observed interfacial disturbances are induced and therefore the mechanism by which mass transfer rates are increased beyond the level predicted by molecular diffusional theory alone. A full account of this work will be published in due course.

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NONSIMILAR LAMINAR NATURAL CONVECTION IN A THERMALLY STRATIFIED FLUID

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NOMENCLATURE

a,	ambient temperature gradient, dT_∞/dx;
C_f,	local skin-friction coefficient;
F,	dimensionless stream function;
F_w,	mass transfer parameter;
$F''(\xi, 0)$,	surface skin-friction parameter;
g,	gravitational acceleration;
G,	dimensionless temperature;
$G'(\xi, 0)$,	surface heat-transfer parameter;
Gr_x,	local Grashof number;
k,	thermal conductivity;
Nu,	local Nusselt number;
Pr,	Prandtl number;
q,	local heat-transfer rate per unit area;
T,	temperature;
T_∞,	ambient temperature;
$T_{\infty 0}$,	ambient temperature at $x = 0$;
x, y,	distances along and perpendicular to the surface.

Greek symbols

β,	bulk coefficient of thermal expansion;
η, ξ,	transformed co-ordinates;
ν,	kinematic viscosity;
ρ,	density;
τ,	shear stress at the surface;
ψ,	dimensional stream function.

Superscript

$'$,	differentiation with respect to η.

Subscripts

w,	condition at the surface;
ξ,	derivative with respect to ξ.

1. INTRODUCTION

THE PROBLEMS of natural convection heat transfer from bodies immersed in fluids do not always admit similarity solutions. The nonsimilarity in this case arises either due to thermally stratified fluid (in which the body is immersed) or due to non-uniform surface temperature of the body. Stratified fluid occurs quite frequently in nature (for example in the atmosphere and ocean) as well as in a number of engineering devices. Although the effect of stratification on the heat removal processes is important, only a few studies have been reported in the literature. Eichhorn [1] studied the effect of linear thermal stratification on the heat transfer of a vertical plate and obtained solutions for three terms in the series expansions of the partial differential equations. Subsequently, Fujii *et al.* [2] considered the effect of non-linear thermal stratification on the foregoing problem and obtained solutions for four terms of the series. Recently, Chen and Eichhorn [3] re-studied the problem of [1] and obtained the solution using the local nonsimilarity method developed by

Sparrow and co-workers [4–6]. The local nonsimilarity method has its own drawbacks as the derivatives of certain terms are discarded in order to reduce the partial differential equations to ordinary differential equations. The method of series expansion is not expected to be valid for large ξ. Hence, for accurate prediction of the heat-transfer rate and skin friction, it is necessary to use an exact method such as a finite-difference scheme.

The aim of this communication is to study the foregoing problem taking into account the effect of mass transfer (which was not considered by previous investigators) by using a new implicit finite-difference scheme developed by Keller and Cebeci [7–9] and to compare the results with those of series expansion and local nonsimilarity methods used by previous investigators [1, 3].

2. GOVERNING EQUATIONS

Let us consider an isothermal vertical finite plate immersed in a stable thermally stratified fluid. The wall temperature is considered as uniform at T_w and the ambient temperature T_∞ is assumed to vary linearly with distance from the leading edge. The natural convection boundary-layer equations in dimensionless form governing the steady incompressible flow for a vertical plate with mass transfer can be expressed as [3]:

$$F''' + 3FF'' - 2F'^2 + G = 4\xi(F'F'_\xi - F''F_\xi), \quad (1)$$

$$Pr^{-1}G'' + 3FG' - 4\xi F' = 4\xi(F'G_\xi - G'F_\xi). \quad (2)$$

The boundary conditions are given by

$$F(\xi,0) = F_w,\ F'(\xi,0) = G(\xi,0) - 1 + \xi = 0, \quad (3)$$
$$\text{and } F'(\xi,\infty) = G(\xi,\infty) = 0,$$

where

$$\left.\begin{aligned} \xi &= ax/\Delta T,\ \eta = [(g\beta\Delta T)(4xv^2)^{-1}]^{1/4}y, \\ \psi(x,y) &= 4v[(g\beta\Delta T)(4v^2)^{-1}]^{1/4}x^{3/4}F(\xi,\eta), \end{aligned}\right\} \quad (4a)$$

$$\left.\begin{aligned} G(\xi,\eta) &= (T - T_\infty)/\Delta T,\ \Delta T = T_w - T_{\infty 0}, \\ T_\infty &= T_{\infty 0} + ax,\ a = dT_\infty/dx > 0, \end{aligned}\right\} \quad (4b)$$

$$\left.\begin{aligned} F_w &= -(4\xi)^{-3/4}Gr_x^{-1/4}\int_0^\xi [(v)_w/(v/x)]\xi^{-1/4}d\xi, \\ Gr_x &= g\beta\Delta T x^3 v^{-2}. \end{aligned}\right\} \quad (4c)$$

where $a > 0$ implies a stably stratified ambient fluid. If the normal velocity at the wall $v_w/(v/x)$ is taken as constant, then F_w = constant ($F_w > 0$ for suction and $F_w < 0$ for injection).

The skin-friction coefficient and heat-transfer coefficient (Nusselt number) can be expressed as [3]

$$\left.\begin{aligned} C_f &= \tau_w/[\rho(v/x)^2] = 4(Gr_x/4)^{3/4}F''(\xi,0), \\ Nu &= qx/k\Delta T = -(Gr_x/4)^{1/4}G'(\xi,0). \end{aligned}\right\} \quad (5)$$

3. RESULTS AND DISCUSSION

Equations (1) and (2) have been solved numerically under conditions (3) using an implicit finite-difference scheme developed by Keller and Cebeci [7–9]. Since the complete description of the method and its application to boundary-layer problems are given in [7–10], the description of the method is not presented here. For the problem under consideration, we have taken the step size $\Delta\eta = 0.05$ and $\Delta\xi = 0.1$. Also the edge of the boundary layer η_∞ has been taken between 5 and 16 depending on the values of ξ and Pr. Further reduction in the step size changes the results only in the 4th decimal place. In order to determine the accuracy of the present method for the problem under consideration, the results of the similarity equations obtained by putting $\xi = 0$ in equations (1)–(3) have been compared with those tabulated in [1] and they are found to agree at least up to the 4th decimal place.

The skin-friction and heat-transfer results ($F''(\xi,0)$, $G'(\xi,0)$) for $F_w = 0$ (i.e. without mass transfer) obtained by the finite-difference method have been compared with those of the series and local nonsimilarity methods [1, 3]. Since the nature of the variation of the skin-friction results is the same as the heat-transfer results, for the sake of brevity, the comparison of only heat-transfer results is shown in Fig. 1. For large Pr ($Pr = 6$), the local nonsimilarity results are found to be in very good agreement with the finite-difference results even for large ξ. However, for small Pr($Pr = 0.7$) and large ξ, they differ from those of the finite-difference scheme. On the other hand, the series solution results are in good agreement with the finite-difference results only for small ξ, but for large ξ, they differ considerably whatever may be the values of Pr and this difference increases as ξ increases. Furthermore, the local nonsimilarity method gives more accurate results than the series solution method. Therefore, for accurate prediction of the heat-transfer and skin-friction results for all values of Pr and ξ, exact method such as a finite-difference method has to be used. As expected, suction increases the heat transfer or skin friction whereas injection does the reverse.

The velocity and temperature profiles for $F_w = 0$ are shown in Figs. 2 and 3 which also contain the corresponding profiles obtained by the local nonsimilarity method [1] and they are

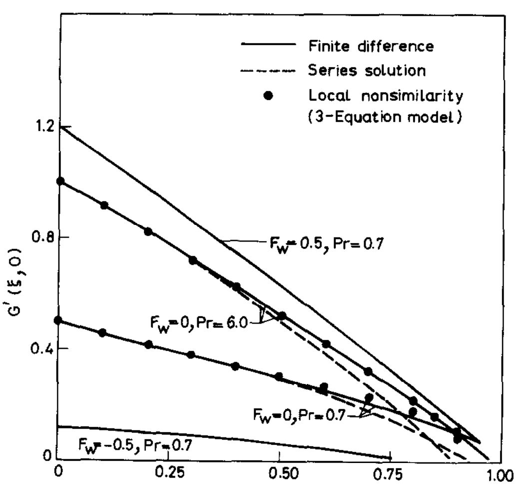

FIG. 1. Heat-transfer results.

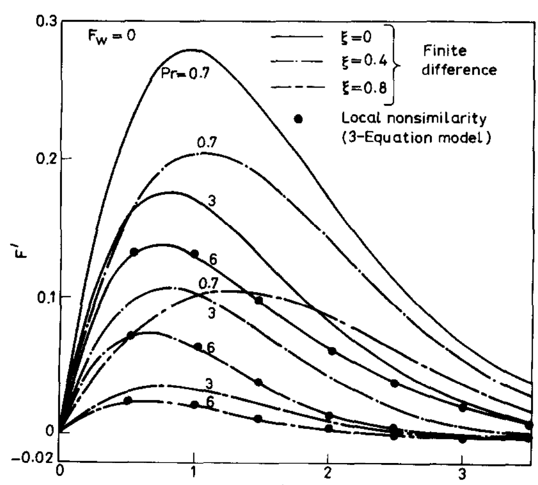

FIG. 2. Velocity profiles ($F_w = 0$).

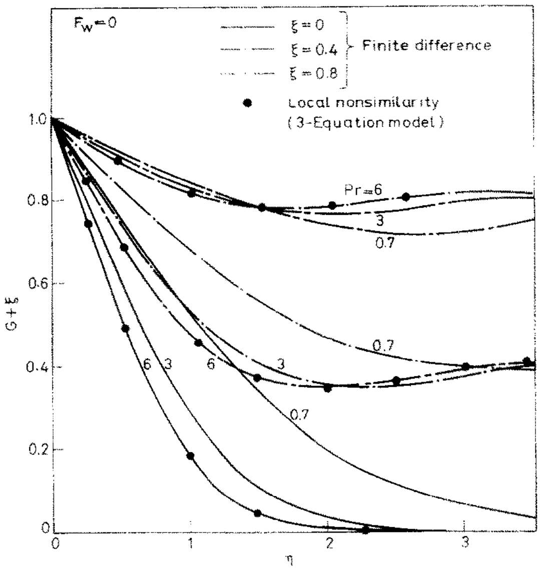

FIG. 3. Temperature profiles ($F_w = 0$).

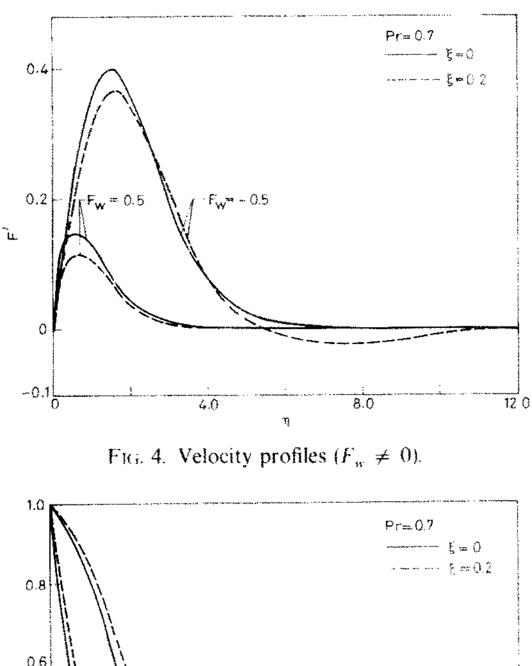

FIG. 4. Velocity profiles ($F_w \neq 0$).

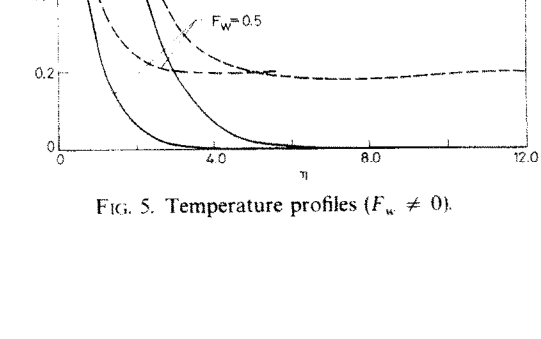

FIG. 5. Temperature profiles ($F_w \neq 0$).

found to be in good agreement except for large ξ. For $\xi \geqslant 0.2$, the velocity profiles show a weak reversal of flow in the outer portion of the boundary layer (see Fig. 2) and there is also a temperature inversion (Fig. 3). These hold good whatever may be the values of the Prandtl number Pr. For small values of Pr (i.e. for $Pr = 0.7, 3.0$), the reversal of flow occurs for large values of η_{∞} (not shown in Fig. 2). Similar effects have also been observed by Eichhorn [1] and Fujii [2]. The effect of mass transfer on the velocity and temperature profiles is displayed in Figs. 4 and 5. It is evident from the figures that the magnitude of flow reversal or temperature inversion increases due to injection and suction does the reverse.

4. CONCLUDING REMARKS

The local nonsimilarity results differ from those of the finite-difference method only when the Prandtl number is small and the longitudinal distance is large, but the series solution results differ considerably for large ξ whatever may be the values of the Prandtl number. Also, the local nonsimilarity method gives better results than the series solution method. The heat transfer and skin friction increase due to suction and the effect of injection is just the reverse. Like previous investigators our study without mass transfer also reveals a temperature inversion and a weak reverse flow in the outer portion of the boundary layer, but the magnitude of the reverse flow and temperature inversion increases due to injection.

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